

İki Boyutlu Projektif Dönüşüm

$$X = \frac{a_{11}x + a_{12}y + a_{13}}{a_{31}x + a_{32}y + 1} \quad i = 1, 2, 3, 4$$

$$Y = \frac{a_{21}x + a_{22}y + a_{23}}{a_{31}x + a_{32}y + 1}$$

$$\begin{aligned} a_{11}x_i + a_{12}y_i + a_{13} - a_{31}x_iX_i - a_{32}y_iX_i &= X_i \\ a_{21}x_i + a_{22}y_i + a_{23} - a_{31}x_iY_i - a_{32}y_iY_i &= Y_i \end{aligned}$$

Koşul denklemleri, $a_{11}, a_{12}, \dots, a_{23}$ katsayıları 8 tanedir ve bu nedenle 4 ortak noktanın (X,Y) ve (x,y) koordinatları yardımıyla çözülür.

Eğer ortak nokta sayısı 4'den fazla ise bilinmeyenler dengelemeli olarak iteratif çözümle bulunur. İlk olarak,

$$1- \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2- Düzeltme denklemleri,
 $i = 1, 2, \dots, n$

$$N_i = \frac{1}{a_{31}x_i + a_{32}y_i + 1}$$

$$V_{xi} = \frac{x_i}{N_i} da_{11} + \frac{y_i}{N_i} da_{12} + \frac{1}{N_i} da_{13} - \frac{x_iX_i}{N_i} da_{31} - \frac{y_iX_i}{N_i} da_{32} - \frac{l_{xi}}{N_i}$$

$$V_{yi} = \frac{x_i}{N_i} da_{21} + \frac{y_i}{N_i} da_{22} + \frac{1}{N_i} da_{23} - \frac{x_iY_i}{N_i} da_{31} - \frac{y_iY_i}{N_i} da_{32} - \frac{l_{yi}}{N_i}$$

$$-l_{xi} = a_{11}x_i + a_{12}y_i + a_{13} - a_{31}x_iX_i - a_{32}y_iX_i - X_i$$

$$-l_{yi} = a_{21}x_i + a_{22}y_i + a_{23} - a_{31}x_iY_i - a_{32}y_iY_i - Y_i$$

3- Normal denklemlerin kurulması ve çözümü

$$V = AX - L$$

$$X = (A^T P A)^{-1} A^T P L$$

$$V = \begin{bmatrix} V_{x1} \\ V_{y1} \\ \vdots \\ V_{xn} \\ V_{yn} \end{bmatrix}, \quad L = \begin{bmatrix} L_{x1} \\ L_{y1} \\ \vdots \\ L_{xn} \\ L_{yn} \end{bmatrix}, \quad X = \begin{bmatrix} da_{11} \\ da_{12} \\ da_{13} \\ da_{21} \\ da_{22} \\ da_{23} \\ da_{31} \\ da_{32} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{x_1}{N_1} & \frac{y_1}{N_1} & \frac{1}{N_1} & 0 & 0 & 0 & -\frac{x_1 X_1}{N_1} & -\frac{y_1 X_1}{N_1} \\ 0 & 0 & 0 & \frac{x_1}{N_1} & \frac{y_1}{N_1} & \frac{1}{N_1} & -\frac{x_1 Y_1}{N_1} & -\frac{y_1 Y_1}{N_1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{x_n}{N_n} & \frac{y_n}{N_n} & \frac{1}{N_n} & 0 & 0 & 0 & -\frac{x_n X_n}{N_n} & -\frac{y_n X_n}{N_n} \\ 0 & 0 & 0 & \frac{x_n}{N_n} & \frac{y_n}{N_n} & \frac{1}{N_n} & -\frac{x_n Y_n}{N_n} & -\frac{y_n Y_n}{N_n} \end{bmatrix}$$

4- İterasyonla çözüm,

$$\begin{aligned} a_{11}^{k+1} &= a_{11}^k + da_{11} & , & & a_{12}^{k+1} &= a_{12}^k + da_{12} & , & & a_{13}^{k+1} &= a_{13}^k + da_{13} \\ a_{21}^{k+1} &= a_{21}^k + da_{21} & , & & a_{22}^{k+1} &= a_{22}^k + da_{22} & , & & a_{23}^{k+1} &= a_{23}^k + da_{23} \\ a_{31}^{k+1} &= a_{31}^k + da_{31} & , & & a_{32}^{k+1} &= a_{32}^k + da_{32} \end{aligned}$$

5- Bilinmeyenler değişmeyene kadar 1-4 işlem adımları tekrarlanır.